Advance Higher Unit 1 — Homework 1 — Solutions

1(a) $\binom{n}{2} = 21$

$$\frac{n!}{2!(n-2)!} = 21$$

$$\frac{n!}{(n-2)!} = 42$$

$$n(n-1) = 42$$

$$n^2 - n - 42 = 0$$

$$(n+6)(n-7) = 0$$

$$\underline{\underline{n = 7}}, \ \cancel{-6}^{\text{NA}}$$

(b) $\binom{n}{n-2} = 45$

$$\frac{n!}{(n-2)!\,[n-(n-2)]!} = 45$$

$$\frac{n!}{(n-2)!\,2!} = 45$$

$$n(n-1) = 90$$

$$n^2 - n - 90 = 0$$

$$(n+9)(n-10) = 0$$

$$n = \cancel{-9}^{\text{NA}}, \ \underline{\underline{10}}$$

(c)
$$\binom{n^2}{n^2-1} = \frac{(n^2)!}{(n^2-1)!\,[n^2-(n^2-1)]!}$$

$$= \frac{(n^2)!}{(n^2-1)!\,1!}$$

$$= \underline{\underline{n^2}}$$

(d) LS $= \binom{n+1}{r}$

$$= \frac{(n+1)!}{r!\,(n+1-r)!}$$

RS $= \binom{n}{r-1} + \binom{n}{r}$

$$= \frac{n!}{(r-1)!\,(n-r+1)!} + \frac{n!}{r!\,(n-r)!}$$

$$= \frac{n!}{(r-1)!\,(n-r+1)(n-r)!} + \frac{n!}{r(r-1)!\,(n-r)!}$$

$$= \frac{r\,n! + n!(n-r+1)}{r(r-1)!\,(n-r+1)(n-r)!}$$

$$= \frac{n!\,(n+1)}{r(r-1)!\,(n-r+1)(n-r)!}$$

$$= \frac{(n+1)!}{r!\,(n+1-r)!} = \underline{\underline{\text{LS}}} \quad \text{QED}$$

2(a) $(p+q)^5$

$$= \binom{5}{0}p^5q^0 + \binom{5}{1}p^4q^1 + \binom{5}{2}p^3q^2 + \binom{5}{3}p^2q^3 + \binom{5}{4}p^1q^4 + \binom{5}{5}p^0q^5$$

$$= \underline{\underline{p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5}}$$

(b) $\left(x - \frac{1}{x}\right)^4$

$$= \binom{4}{0}x^4\left(\frac{1}{x}\right)^0 + \binom{4}{1}x^3\left(-\frac{1}{x}\right)^1 + \binom{4}{2}x^2\left(-\frac{1}{x}\right)^2 + \binom{4}{3}x^1\left(-\frac{1}{x}\right)^3 + \binom{4}{4}x^0\left(-\frac{1}{x}\right)^4$$

$$= \underline{\underline{x^4 - 4x^2 + 6 - \frac{4}{x^2} + \frac{1}{x^4}}}$$

(c) x^5 term from $(1-2x)^7$

$$\binom{7}{5}1^2(-2x)^5 = 21.1.(-2x)^5 = -672x^5$$

Coefficient of x^5 is $\underline{\underline{-672}}$

(d) $\left(1 - \frac{3}{2}x - x^2\right)^5 = \left[1 - \frac{3}{2}x\left(1 + \frac{2}{3}x\right)\right]^5$

$$= \binom{5}{0}1^5\left[-\tfrac{3}{2}x\left(1+\tfrac{2}{3}x\right)\right]^0 + \binom{5}{1}1^4\left[-\tfrac{3}{2}x\left(1+\tfrac{2}{3}x\right)\right]^1 + \binom{5}{2}1^3\left[-\tfrac{3}{2}x\left(1+\tfrac{2}{3}x\right)\right]^2$$

$$+ \binom{5}{3}1^2\left[-\tfrac{3}{2}x\left(1+\tfrac{2}{3}x\right)\right]^3 + \binom{5}{4}1^1\left[-\tfrac{3}{2}x\left(1+\tfrac{2}{3}x\right)\right]^4 + \binom{5}{5}1^0\left[-\tfrac{3}{2}x\left(1+\tfrac{2}{3}x\right)\right]^5$$

$$= 1 + 5.-\tfrac{3}{2}x\left(1+\tfrac{2}{3}x\right) + 10.\left(-\tfrac{3}{2}x\right)^2\left(1+\tfrac{2}{3}x\right)^2 + 10.\left(-\tfrac{3}{2}x\right)^3\left(1+\tfrac{2}{3}x\right)^3$$

$$+ 5.\left(-\tfrac{3}{2}x\right)^4\left(1+\tfrac{2}{3}x\right)^4 + \left(-\tfrac{3}{2}x\right)^5\left(1+\tfrac{2}{3}x\right)^5$$

$$= 1 - \frac{15}{2}x\left(1+\tfrac{2}{3}x\right) + \frac{45}{2}x^2\left(1+\tfrac{2}{3}x\right)^2 - \frac{135}{4}x^3\left(1+\tfrac{2}{3}x\right)^3$$

$$+ \frac{405}{16}x^4\left(1+\tfrac{2}{3}x\right)^4 + \text{-----}$$ NOT NEEDED.

$$= 1 - \frac{15}{2}x - 5x^2 + \frac{45}{2}x^2\left(1 + \frac{4}{3}x + \frac{4}{9}x^2\right)$$

$$- \frac{135}{4}x^3\left(1 + 2x + \frac{4}{3}x^2 + \frac{8}{27}x^3\right) + \frac{405}{16}x^4\left(1 + \frac{8}{3}x + \frac{8}{3}x^2 + \frac{32}{27}x + \frac{16}{81}\right.$$

2(d) continued

$$= 1 - \frac{15}{2}x + \frac{35}{2}x^2 - \frac{15}{4}x^3 - \frac{515}{16}x^4 + \text{------}$$

3(a) $(1.1)^5 = (1+0.1)^5$

$$= \binom{5}{0}1^5(0.1)^0 + \binom{5}{1}1^4(0.1)^1 + \binom{5}{2}1^3(0.1)^2 + \binom{5}{3}1^2(0.1)^3 + \binom{5}{4}1^1(0.1)^4 + \binom{5}{5}1^0(0.1)^5$$

$$= 1 + 5(0.1) + 10(0.01) + 10(0.001) + 5(0.0001) + 0.00001$$

$$= 1 + 0.5 + 0.1 + 0.01 + 0.0005 + 0.00001$$

$$= \underline{\underline{1.61}} \quad \text{(to 3 sig figs)}$$

(b) $(0.98)^4 = (1-0.02)^4$

$$= \binom{4}{0}1^4(-0.02)^0 + \binom{4}{1}1^3(-0.02)^1 + \binom{4}{2}1^2(-0.02)^2 + \binom{4}{3}1^1(-0.02)^3 + \binom{4}{4}1^0(-0.02)^4$$

$$= 1 - 4.(0.02) + 6(0.0004) - 4(0.000008) + 0.00000016$$

$$= 1 - 0.08 + 0.0024 - 0.000032 + \text{-----}$$

$$= \underline{\underline{0.922}} \quad \text{(to 3 sig figs)}$$

4. $\left(3x^2 - \frac{1}{2x}\right)^9$

$$= \binom{9}{0}(3x^2)^9 . \left(-\frac{1}{2x}\right)^0 + \binom{9}{1}(3x^2)^8\left(-\frac{1}{2x}\right)^1 + \text{-----}$$

For term independant of x: $\binom{9}{6}(3x^2)^3\left(-\frac{1}{2x}\right)^6$

$$\frac{x^6}{x^6} = x^0 = 1$$

$$= 84.27x^6\left(\frac{1}{64x^6}\right)$$

$$= \frac{2268}{64}$$

$$= \underline{\underline{\frac{567}{16}}}$$

5(a) Let $\dfrac{5x+5}{x^2+3x-4} = \dfrac{A}{(x+4)} + \dfrac{B}{(x-1)}$

$$\frac{5x+5}{(x+4)(x-1)} = \frac{A}{x+4} + \frac{B}{x-1}$$

$$5x+5 = A(x-1) + B(x+4)$$

$x=1 \Rightarrow 10 = 5B \Rightarrow B=2$

$x=-4 \Rightarrow -15 = -5A \Rightarrow A=3$

$$\therefore \frac{5x+5}{x^2+3x-4} = \underline{\underline{\frac{3}{x+4} + \frac{2}{x-1}}}$$

(b) $\dfrac{x^3}{x^2-3x+2}$

$$\begin{array}{r|l} & x+3 \\ \hline x^2-3x+2 & x^3 \\ & x^3-3x^2+2x \\ \hline & 3x^2-2x \\ & 3x^2-9x+6 \\ \hline & 7x-6 \end{array}$$

$= x+3+\dfrac{7x-6}{x^2-3x+2}$

Let $\dfrac{7x-6}{(x-2)(x-1)} = \dfrac{A}{x-2} + \dfrac{B}{x-1}$

$$7x-6 = A(x-1) + B(x-2)$$

$x=1 \Rightarrow 1 = -B \Rightarrow B=-1$

$x=2 \Rightarrow 8 = A \Rightarrow A=8$

$$\frac{x^3}{x^2-3x+2} = \underline{\underline{x+3+\frac{8}{x-2} - \frac{1}{x-1}}}$$

5(c) $\dfrac{3x+1}{(x-1)(x^2-1)}$

$$\Rightarrow \frac{3x+1}{(x-1)(x-1)(x+1)} = \frac{A}{x+1} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$$

$$3x+1 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$$

$x=1 \Rightarrow \quad 4 = 2C \quad \Rightarrow C = 2$

$x=0 \Rightarrow \quad 1 = A - B + 2 \quad \Rightarrow A - B = -1$

$x=-1 \Rightarrow \quad -2 = 4A \quad \Rightarrow A = -\tfrac{1}{2}, \ B = \tfrac{1}{2}$

$$\frac{3x+1}{(x-1)(x^2-1)} = \underline{\underline{\frac{2}{(x-1)^2} + \frac{1}{2(x-1)} - \frac{1}{2(x+1)}}}$$

(d) $\dfrac{3x+3}{(x-1)(x^2+x+1)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+x+1}$

$$3x+3 = A(x^2+x+1) + (Bx+C)(x-1)$$

$x=1 \Rightarrow 6 = 3A \Rightarrow \underline{\underline{A=2}}$

$\therefore\ 3x+3 = 2(x^2+x+1) + (Bx+C)(x-1)$

$x=0 \Rightarrow 3 = 2 - C \Rightarrow \underline{\underline{C=-1}}$

$\therefore\ 3x+3 = 2(x^2+x+1) + (Bx-1)(x-1)$

$x=-1 \Rightarrow 0 = 2 + (-B-1).(-2)$

$\quad = 2 + 2B + 2 \Rightarrow \underline{\underline{B=-2}}$

$$\frac{3x+3}{(x-1)(x^2+x+1)} = \underline{\underline{\frac{2}{x-1} - \frac{2x+1}{x^2+x+1}}}$$