

Higher Maths 2005

Paper 1 Solutions

1. $m = \tan 60^\circ$
 $m = \sqrt{3}$

$$y - b = m(x - a)$$

$$y - 0 = \sqrt{3}(x - (-2))$$

$$y = \sqrt{3}x + 2\sqrt{3}$$

2. (a) Congruent circles \Rightarrow radii equal

P = midpoint of AB

From equations; A(-3, -2) and B(3,6)

\Rightarrow P(0,2)

(b) $AB = \sqrt{(3 - (-3))^2 + (6 - (-2))^2}$

$$= \sqrt{(6)^2 + (8)^2}$$

$$= \sqrt{100}$$

= 10 units

3. (a) $\overrightarrow{DF} = \frac{2}{3}\overrightarrow{DB}$

$$\overrightarrow{DF} = \frac{2}{3} \begin{pmatrix} 6 \\ 3 \\ -9 \end{pmatrix}$$

$$\overrightarrow{DF} = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix}$$

\Rightarrow F(10,5,3)

(b) $\overrightarrow{AF} = \overrightarrow{AD} + \overrightarrow{DF}$

$$\overrightarrow{AF} = \begin{pmatrix} -6 \\ 3 \\ 9 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix}$$

$$\overrightarrow{AF} = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$$

4. (a) $h(x) = g(f(x))$

$$= g(3x - 1)$$

$$= (3x - 1)^2 + 7$$

(b) (i) Minimum turning point at $(\frac{1}{3}, 7)$

(ii) Range: $y \geq 7$

5. $\frac{d}{dx}(1 + 2\sin x)^4 = 4(1 + 2\sin x)^3 \cdot 2\cos x$
 $= 8\cos x(1 + 2\sin x)^3$

6. (a) $u_{n+1} = ku_n + 5$

$$L = kL + 5$$

$$4 = 4k + 5$$

$$-1 = 4k$$

$$-\frac{1}{4} = k$$

(b) (i) $u_{n+1} = mu_n + 5$ $u_0 = 3$

$$u_1 = 3m + 5$$

$$u_2 = m(3m + 5) + 5$$

$$u_2 = 3m^2 + 5m + 5$$

(ii) $3m^2 + 5m + 5 = 7$

$$3m^2 + 5m - 2 = 0$$

$$(3m - 1)(m + 2) = 0$$

$$\Rightarrow m = \frac{1}{3} \text{ or } m = -2$$

$m = -2$ produces a sequence with no limit
as $-1 < m < 1$ for a limit to exist

7. (a) $a = 4, b = 5$

(b) Domain: $x > 4$

8. (a)

3	2	-7	0	9
		6	-3	-9
	2	-1	-3	0

$\Rightarrow (x - 3)$ is a factor

Quotient: $2x^2 - x - 3$

$$\Rightarrow f(x) = (x - 3)(2x^2 - x - 3)$$

$$= (x - 3)(x + 1)(2x - 3)$$

(b) $f(x)$ crosses x -axis when $y = 0$

$$\Rightarrow x = -1, \frac{3}{2}, 3$$

$$\Rightarrow (-1, 0) \quad (\frac{3}{2}, 0) \quad (3, 0)$$

$f(x)$ crosses y -axis when $x = 0$

$$\Rightarrow y = 9$$

$$\Rightarrow (0, 9)$$

8. (c) $f'(x) = 6x^2 - 14x$

For stationary points, $f'(x) = 0$

$$\Rightarrow 6x^2 - 14x = 0$$

$$2x(3x - 7) = 0$$

$$\Rightarrow x = 0 \text{ or } x = \frac{7}{3}$$

when $x = 0, y = 9$

$$\text{when } x = \frac{7}{3}, y = 2\left(\frac{7}{3}\right)^3 - 7\left(\frac{7}{3}\right) + 9$$

$$= \frac{686}{27} - \frac{343}{9} + 9$$

$$= \frac{686}{27} - \frac{1029}{27} + \frac{243}{27}$$

$$= \frac{-100}{27}$$

x	→	0	→	$\frac{7}{3}$	→
$f'(x)$	+	0	-	0	+
	/	-	\	-	/

\Rightarrow Max TP at (0,9)

\Rightarrow Min TP at $\left(\frac{7}{3}, \frac{-100}{27}\right)$

$$\text{when } x = -2, y = 2(-2)^3 - 7(-2) + 9$$

$$= -16 - 28 + 9$$

$$= -35$$

$$(-2, -35)$$

$$y = 2(2)^3 - 7(2) + 9$$

$$= 16 - 28 + 9$$

$$= -3$$

$$(2, -3)$$

\Rightarrow greatest value of f in the interval $-2 \leq x \leq 2$ is 9 and least value of f is -35

9. $\cos 2x = \frac{7}{25}$ $\cos 2x = \frac{7}{25}$

$2\cos^2 x - 1 = \frac{7}{25}$ $1 - 2\sin^2 x = \frac{7}{25}$

$2\cos^2 x = \frac{32}{25}$ $-2\sin^2 x = -\frac{18}{25}$

$\cos^2 x = \frac{16}{25}$ $\sin^2 x = \frac{9}{25}$

$\cos x = \pm \frac{4}{5}$ $\sin x = \pm \frac{3}{5}$

$\Rightarrow \cos x = \frac{4}{5}$ and $\sin x = \frac{3}{5}$ for $0 < x < \frac{\pi}{2}$ (only 1st quadrant solutions required)

10. (a)

$$\sin x - \sqrt{3} \cos x$$

$$k \sin(x - a) = k \sin x \cos a - k \cos x \sin a$$

$$k \cos a = 1$$

$$k = \sqrt{(1)^2 + (\sqrt{3})^2}$$

$$\tan a = \frac{k \sin a}{k \cos a}$$

$$k \sin a = \sqrt{3}$$

$$k = \sqrt{1+3}$$

$$\tan a = \frac{\sqrt{3}}{1}$$

$$k = \sqrt{4}$$

$$a = \frac{\pi}{3}$$

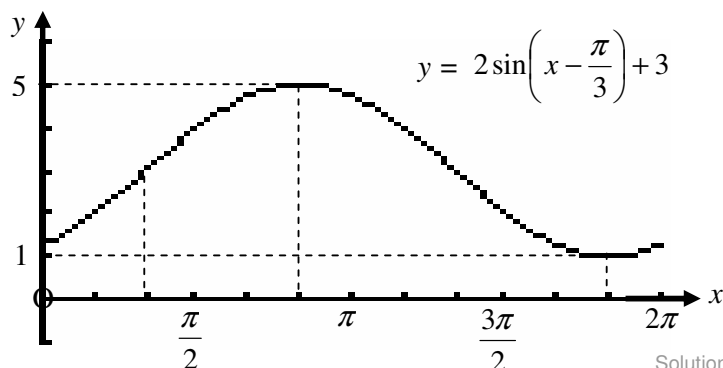
$$k = 2$$

a is in 1st quadrant so $a = \frac{\pi}{3}$

S	A
✓	✓✓✓✓
T	C
✓	✓

$$\Rightarrow \sin x - \sqrt{3} \cos x = 2 \sin\left(x - \frac{\pi}{3}\right)$$

(b)



$$11. \text{ (a) } (x-t)^2 + (y-0)^2 = (2)^2$$

$$(x-t)^2 + y^2 = 4$$

$$\text{(b) } (x-t)^2 + (2x)^2 = 4$$

$$x^2 - 2tx + t^2 + 4x^2 = 4$$

$$5x^2 - 2tx + t^2 - 4 = 0$$

If $y = 2x$ is tangent then $b^2 - 4ac = 0$

$$(-2t)^2 - 4(5)(t^2 - 4) = 0$$

$$4t^2 - 20t^2 + 80 = 0$$

$$80 - 16t^2 = 0$$

$$80 = 16t^2$$

$$5 = t^2$$

$$t = \pm\sqrt{5}$$

$\Rightarrow t = \sqrt{5}$ as t is on positive x -axis in diagram