HIGHER 2007 PAPER 1 SOLUTIONS

1. $P(-1,4)$

$3x - y + 2 = 0$

$y = 3x + 2$

$\Rightarrow m = 3$

$y - b = m(x - a)$

$y - 4 = 3(x + 1)$

$y - 4 = 3x + 3$

$y = 3x + 7$

2.

$(-2,1,-1)$ $(1,3,2)$

$BC = 2AB$

$\vec{AC} = 3\vec{AB}$

$= 3\begin{pmatrix}3\\2\\3\end{pmatrix}$

$= \begin{pmatrix}9\\6\\9\end{pmatrix}$ $\Rightarrow$ $C(7,7,8)$

3. $f(x) = x^2 + 1$ $g(x) = 1 - 2x$

a) $g(f(x)) = g(x^2+1)$

$= 1 - 2(x^2+1)$

$= 1 - 2x^2 - 2$

$= -1 - 2x^2$

b) $g(g(x)) = g(1-2x)$

$= 1 - 2(1-2x)$

$= 1 - 2 + 4x$

$= 4x - 1$

4. $kx^2 - x - 1 = 0$

$a = k \quad b = -1 \quad c = -1$

for no real roots $b^2 - 4ac < 0$

$$b^2 - 4ac = (-1)^2 - 4(k)(-1)$$

$$= 1 + 4k$$

$$\Rightarrow \quad 1 + 4k < 0$$

$$4k < -1$$

$$k < -\frac{1}{4}$$

5. $x^2 + y^2 - 14x - 16y + 77 = 0$

$C(7, 8) \qquad r = \sqrt{49 + 64 - 77}$

$$r = \sqrt{36}$$

$$r = 6$$

$\Rightarrow$ radius of circle D = 2

Centre of B equal to large circle, ie (7,8)

Centre of D is equivalent to 4 radii to the right, ie. 8

$\Rightarrow C_D(15, 8)$

$\Rightarrow$ equation of circle D is $(x-15)^2 + (y-8)^2 = 4$

6. $\sin 2x^\circ = 6\cos x^\circ \qquad 0 \leqslant x \leqslant 360$

$2\sin x \cos x = 6\cos x^\circ$

$2\sin x^\circ \cos x^\circ - 6\cos x^\circ = 0$

$2\cos x^\circ (\sin x^\circ - 3) = 0$

either $2\cos x^\circ = 0$ or $\sin x - 3 = 0$

$x = 90, 270$ $\sin x = 3$

undefined

$\Rightarrow \quad x \in \{90^\circ, 270^\circ\}$

7. $u_{n+1} = \frac{1}{4}u_n + 16 \qquad u_0 = 0$

a) $u_1 = 16 \qquad u_2 = \frac{1}{4}(16) + 16 \qquad u_3 = \frac{1}{4}(20) + 16$

$= 20 \qquad = 21$

b) (i) A limit exists as $-1 < m < 1$, ie $-1 < \frac{1}{4} < 1$

(ii) $L = \frac{c}{1-m}$ or. at limit $L = \frac{1}{4}L + 16$

$L = \frac{16}{1-\frac{1}{4}}$ $\frac{3}{4}L = 16$

$L = \frac{16}{\frac{3}{4}}$ $L = \frac{16}{\frac{3}{4}}$ etc.

$L = \frac{4}{3} \cdot 16$ $\Rightarrow \quad k = \frac{64}{3}$

$L = \frac{64}{3}$

8. a)

3	1	-4	1	6
		3	-3	-6
	1	-1	-2	0

$\Rightarrow x = 3$ is a root

or when $x = 3$,

$y = (3)^3 - 4(3)^2 + (3) + 6$

$y = 27 - 4.9 + 9$

$y = 27 - 36 + 9$

$y = 0$

b) Quotient $= x^2 - x - 2$

$\Rightarrow y = (x-3)(x^2 - x - 2)$

on x-axis, $y = 0$

$\Rightarrow (x-3)(x^2 - x - 2) = 0$

$(x-3)(x-2)(x+1) = 0$

$\Rightarrow$ roots at $x = -1, 2, 3$

$\Rightarrow$ A(2,0) (B(3,0))

c) Shaded area $= \int_0^2 (x^3 - 4x^2 + x + 6)\,dx$

$= \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + 6x\right]_0^2$

$= \left(\frac{(2)^4}{4} - \frac{4(2)^3}{3} + \frac{(2)^2}{2} + 6(2)\right) - 0$

$= \frac{16}{4} - \frac{32}{3} + \frac{4}{2} + 12$

$= 18 - \frac{32}{3}$

$= \frac{54}{3} - \frac{32}{3}$

$= \frac{22}{3}$ units^2

9. a) $f(x) = 3x - x^3$

on x-axis, $y = 0$

$\Rightarrow \quad 3x - x^3 = 0$

$x(3 - x^2) = 0$

$x = 0 \quad$ or $\quad 3 - x^2 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

$(-\sqrt{3}, 0)\ ,\ (0,0)\ ,\ (\sqrt{3}, 0)$

on y-axis, $x = 0$

$\Rightarrow \quad y = 0$

$(0,0)$

b) $f'(x) = 3 - 3x^2$

for stat pts. $f'(x) = 0$

$3 - 3x^2 = 0$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

$f(1) = 3(1) - (1)^3$

$= 3 - 1$

$= 2$

$f(-1) = 3(-1) - (-1)^3$

$= -3 - (-1)$

$= -2$

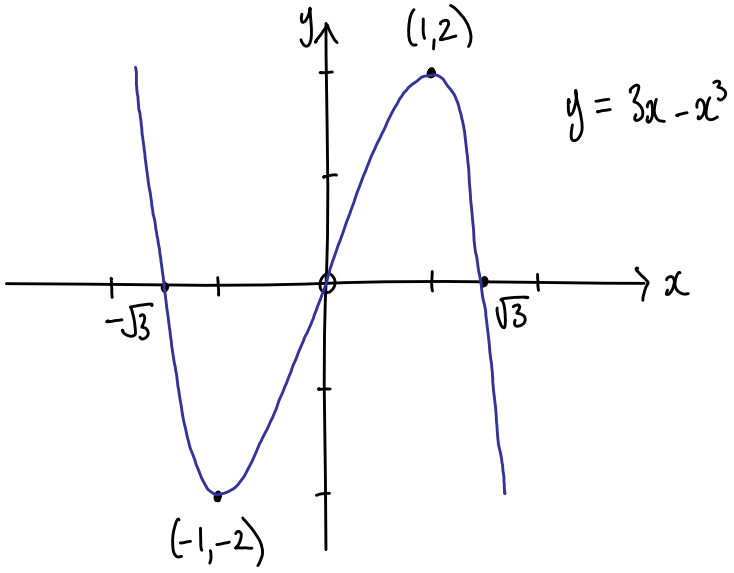
$\Rightarrow$ stat pts at $(-1, -2)$ and $(1, 2)$

x	$\rightarrow$	-1	$\rightarrow$	1	$\rightarrow$
$f'(x)$	$-$	0	$+$	0	$-$
Shape	$\backslash$	—	$/$	—	$\backslash$

Min TP at $(-1, -2)$

Max TP at $(1, 2)$

9. c)



10.

$$y = \sqrt{3x^2+2}$$

$$= (3x^2+2)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}(3x+2)^{-1/2} \cdot 6x$$

$$= 3x(3x+2)^{-1/2}$$

$$= \frac{3x}{\sqrt{3x^2+2}}$$

11. a) $f(x) = \sqrt{3}\cos x + \sin x$

$k\cos(x-\alpha) = k\cos x\cos\alpha + k\sin x\sin\alpha$

$k\cos\alpha = \sqrt{3}$

$k\sin\alpha = 1$

$k^2 = (\sqrt{3})^2 + (1)^2$

$k^2 = 4$

$k = 2$

$\tan\alpha = \dfrac{k\sin\alpha}{k\cos\alpha}$

$\tan\alpha = \dfrac{1}{\sqrt{3}}$

acute angle $= \dfrac{\pi}{6}$

α is in 1st quadrant

$\Rightarrow \alpha = \dfrac{\pi}{6}$

S ✓	A ✓✓
T ✓	C ✓

$\Rightarrow f(x) = 2\cos\left(x - \dfrac{\pi}{6}\right)$

b)