HIGHER MATHS 2009 — PAPER 1 SOLUTIONS

1. $u_{n+1} = 3u_n + 4 \qquad u_1 = 2$

$u_2 = 3(2) + 4$

$= 10$

$u_3 = 3(10) + 4$

$= 34$

A

2. $x^2 + y^2 + 8x + 6y - 75 = 0$

$C(-4, -3)$

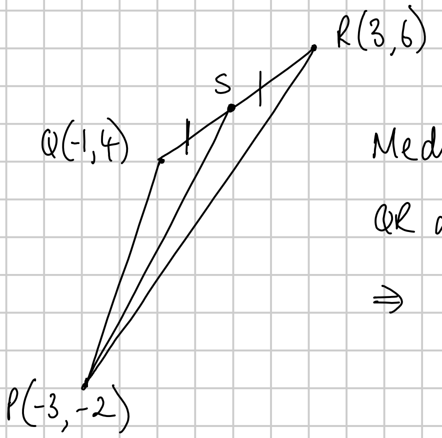
$r = \sqrt{16 + 9 - (-75)}$

$r = \sqrt{100}$

$r = 10$

B

3.



Median PS meets QR at midpoint

$\Rightarrow S(1, 5)$

$m_{PS} = \dfrac{5-(-2)}{1-(-3)}$

$m_{PS} = \dfrac{7}{4}$

D

4. $y = 5x^3 - 12x \qquad (1, -7)$

$\dfrac{dy}{dx} = 15x^2 - 12$

when $x = 1$, $\dfrac{dy}{dx} = 15(1)^2 - 12$

$= 3$

C

5. $S(2, 3) \quad T(5, -1)$

$ST = \sqrt{(5-2)^2 + (-1-3)^2}$

$ST = \sqrt{9 + 16}$

$ST = 5$

$m_{ST} = \dfrac{-1-3}{5-2}$

$m_{ST} = \dfrac{-4}{3}$

B

6. $u_{n+1} = 0.7u_n + 10$

a limit exists as $-1 < 0.7 < 1$

at the limit $L = 0.7L + 10$

(A)

$0.3L = 10$

$L = \frac{10}{0.3}$

$L = \frac{100}{3}$

7. $\cos x = \frac{1}{\sqrt{5}}$

$\cos 2x = 2\cos^2 x - 1$

$= 2\left(\frac{1}{\sqrt{5}}\right)^2 - 1$

(A)

$= 2 \cdot \frac{1}{5} - 1$

$= \frac{2}{5} - 1$

$= -\frac{3}{5}$

8. $\frac{d}{dx}\left(\frac{1}{4x^3}\right)$

$= \frac{d}{dx}\left(\frac{1}{4}x^{-3}\right)$

(D)

$= -\frac{3}{4}x^{-4}$

$= \frac{-3}{4x^4}$

9. $y = 2x \qquad x^2 + y^2 = 5$

for points of intersection $y = y$

$\Rightarrow x^2 + (2x)^2 = 5$

$x^2 + 4x^2 = 5$

(A)

$5x^2 = 5$

$x^2 = 1$

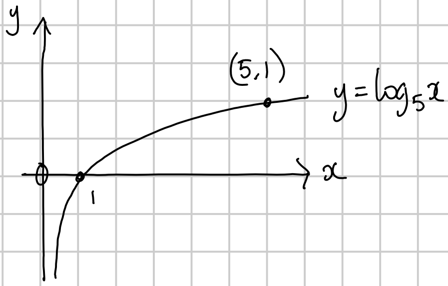
$x = \pm 1$

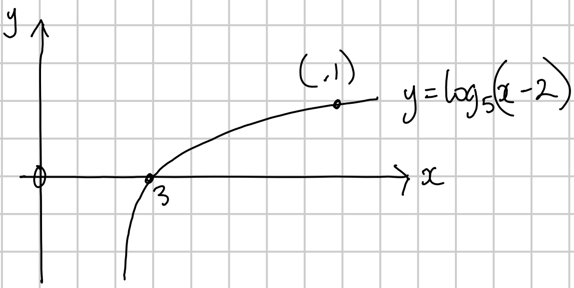
10. $y = \log_5(x-2)$

$\uparrow$

$y = \log_5 x$ moved 2 places right

B





11. $(4\sin x - \sqrt{5})(\sin x + 1) = 0$

either $4\sin x - \sqrt{5} = 0$ or $\sin x + 1 = 0$

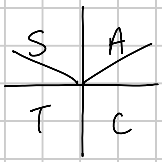
$\sin x = \frac{\sqrt{5}}{4}$ $\qquad$ $\sin x = -1$

$x = \frac{3\pi}{2}$ $\quad$ 1 solution

positive sin value

$\Rightarrow$ 2 solutions

Altogether $\underline{3}$ solutions



B

12. $f(x) = 2x^2 - x - 9$

$b^2 - 4ac = (-1)^2 - 4(2)(-9)$

$= 1 + 72$

$= 73$

C

$b^2 - 4ac > 0 \Rightarrow$ two real roots

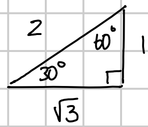
13. $k \sin a^\circ = 1$ $\qquad$ $\tan a^\circ = \frac{k\sin a^\circ}{k\cos a^\circ}$

$k\cos a^\circ = \sqrt{3}$ $\qquad$ $\tan a^\circ = \frac{1}{\sqrt{3}}$

$k^2 = (1)^2 + (\sqrt{3})^2$ $\qquad$ $a^\circ = 30^\circ$

$k^2 = 4$

$k = 2$



B

14. $f(x) = \underbrace{2\sin\left(3x - \frac{\pi}{2}\right)} + 5$

min $= -2$
max $= 2$

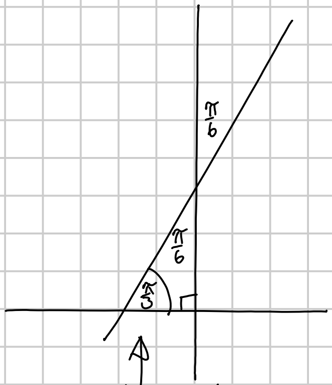
$\Rightarrow$ min $= -2 + 5$
$= 3$

max $= 2 + 5$
$= 7$

$\Rightarrow$ $3 \leqslant f(x) \leqslant 7$

C

15.

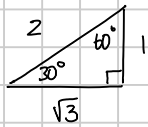


need angle on positive direction of x-axis

$m = \tan\theta$

$m = \tan\frac{\pi}{3}$

$m = \sqrt{3}$



A

16. $-\int_0^1 \left(4x^3 - 9x^2\right) dx$

negative as below x-axis

$= -\left[\frac{4x^4}{4} - \frac{9x^3}{3}\right]_0^1$

$= -\left[x^4 - 3x^3\right]_0^1$

B

17. $\underline{u} = \begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix} = -3\underline{i} + 4\underline{k}$

a unit vector has magnitude equal to 1

$|\underline{u}| = \sqrt{(-3)^2 + (0)^2 + (4)^2}$

$= 5$

A

$\Rightarrow$ unit vector should be $\frac{1}{5}\underline{u}$

$\Rightarrow$ $-\frac{3}{5}\underline{i} + \frac{4}{5}\underline{k}$

18. $f(x) = (4-3x^2)^{-\frac{1}{2}}$ (chain rule)

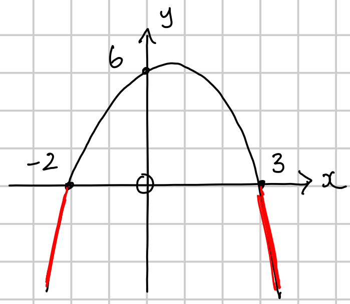
$$f'(x) = -\frac{1}{2}(4-3x^2)^{-\frac{3}{2}} \cdot (-6x)$$

$$= 3x(4-3x^2)^{-\frac{3}{2}}$$

derivative of bracket

D

19. $6 + x - x^2 < 0$



Solve $6 + x - x^2 = 0$ to find roots

$$(3-x)(2+x) = 0$$

$$x = -2, 3$$

$\Rightarrow$ $6 + x - x^2 < 0$ for $x < -2$ and $x > 3$

C

20. $A = 2\pi r^2 + 6\pi r$

$$\frac{dA}{dr} = 4\pi r + 6\pi$$

when $r = 2$,

$$\frac{dA}{dr} = 4\pi(2) + 6\pi$$

$$= 14\pi$$

C

21. (a) $P(-3, 0)$

(b) $m_{QR} = \dfrac{-2-6}{8-4}$

$m_{QR} = \dfrac{-8}{4}$

$m_{QR} = -2$

$\Rightarrow m_{PT} = \frac{1}{2}$ as $m_1 m_2 = -1$ for $\perp$ lines

$P(3,0)$ $m_{PT} = \frac{1}{2}$

$y - 0 = \frac{1}{2}(x+3)$

$y = \frac{1}{2}x + \frac{3}{2}$

(c) $Q(4, 6)$ $m_{QR} = -2$

$y - 6 = -2(x-4)$

$y - 6 = -2x + 8$

$y = -2x + 14$

for points of intersection $y = y$

$\Rightarrow \frac{1}{2}x + \frac{3}{2} = -2x + 14$

$x + 3 = -4x + 28$

$5x = 25$

$x = 5$

$\Rightarrow y = -2(5) + 14$

$y = 4$

$\Rightarrow T(5, 4)$

22. (a) $D(10, -8, -15)$ $E(1, -2, -3)$ $F(-2, 0, 1)$

(pick any two pairs to prove collinearity)

(i) $\vec{DE} = \begin{pmatrix} -9 \\ 6 \\ 12 \end{pmatrix}$ $\vec{EF} = \begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$

$\vec{DE} = 3\begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$

$\vec{DE} = 3\vec{EF}$ $\Rightarrow$ D, E & F are collinear as parallel and share a common point.

$\Rightarrow \vec{DE} \parallel \vec{EF}$

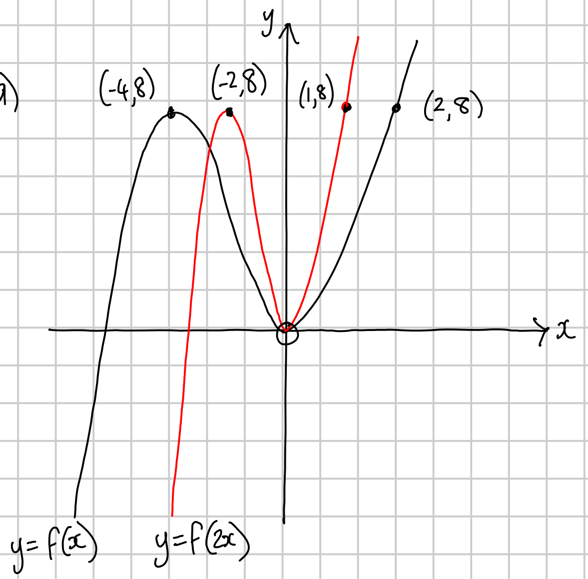
(ii) $\vec{DE} : \vec{EF} = 3 : 1$

(b) $\vec{DE} = \begin{pmatrix} -9 \\ 6 \\ 12 \end{pmatrix}$ $\qquad$ $\vec{GE} = \begin{pmatrix} 1-k \\ -3 \\ -3 \end{pmatrix}$

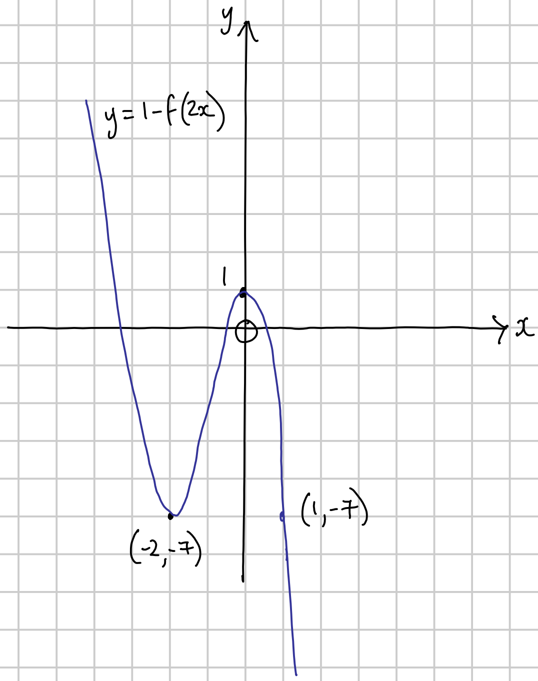
If $\vec{DE} \perp \vec{GE}$ then $\vec{DE} \cdot \vec{GE} = 0$

$$\Rightarrow \quad -9(1-k) + 6(-3) + 12(-3) = 0$$
$$-9 + 9k - 18 - 36 = 0$$
$$9k - 63 = 0$$
$$9k = 63$$
$$k = 7$$

23. (a)



(b)



24. (a)

$$\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$$
$$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}$$
$$= \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} + \frac{1}{2}\cdot\frac{1}{\sqrt{2}}$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$
$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$
$$= \frac{\sqrt{3}+1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$
$$\sin\left(\frac{7\pi}{12}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$$

(b) $\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

$$LHS = \sin(A+B) + \sin(A-B)$$

$$= \sin A\cos B + \cancel{\cos A\sin B} + \sin A\cos B - \cancel{\cos A\sin B}$$

$$= 2\sin A\cos B$$

(c) (i) $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$

$$\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$$

$$= 2\sin\frac{\pi}{3}\cos\frac{\pi}{4}$$

$$= 2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{6}}{2}$$