HIGHER MATHS 2010 – PAPER 1 SOLUTIONS

PART A

1. $2x - 3y - 6 = 0$

$$3y = 2x - 6$$
$$y = \frac{2}{3}x - 2$$
$$m = \frac{2}{3}$$
$$\Rightarrow m_\perp = -\frac{3}{2}$$

A

2. $u_{n+1} = 2u_n + 3 \quad u_0 = 1$

$$u_1 = 2(1) + 3$$
$$= 5$$
$$u_2 = 2(5) + 3$$
$$= 13$$

C

3. $\underline{u} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \quad \underline{v} = \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$

$$3\underline{u} - 2\underline{v} = 3\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} - 2\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$$
$$= \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \\ 8 \end{pmatrix}$$
$$= \begin{pmatrix} 8 \\ -4 \\ -5 \end{pmatrix}$$

D

4. 3 waves in 2π (360°)

amplitude $= 2$

$$\Rightarrow y = 2\cos 3x$$

A

5. $x^2 + 8x + 3$

$$= (x+4)^2 - 16 + 3$$
$$= (x+4)^2 - 13$$
$$\Rightarrow q = -13$$

B

6. $kx^2 - 3x + 2 = 0$

for equal roots $b^2 - 4ac = 0$

$$\Rightarrow (-3)^2 - 4k(2) = 0$$
$$9 - 8k = 0$$
$$8k = 9$$
$$k = \frac{9}{8}$$

D

7. $u_{n+1} = \frac{1}{4}u_n + 7, \quad u_0 = -2$

a limit exists as $-1 < \frac{1}{4} < 1$

at the limit $u_{n+1} = u_n = L$

$\Rightarrow \quad L = \frac{1}{4}L + 7$

$\frac{3}{4}L = 7$

$L = \frac{4}{3}.7$

$L = \frac{28}{3}$ C

8. $x^2 + y^2 - 6x - 10y + 9 = 0$

$c(3,5) \quad r = \sqrt{(3)^2 + (5)^2 - 9}$

$r = \sqrt{9 + 25 - 9}$

$r = \sqrt{25}$

$r = 5$

C is 5 high + radius of 5

$\Rightarrow \quad y = 10$ B

9. $\int (2x^{-4} + \cos 5x)\, dx$

$= \frac{2x^{-3}}{-3} + \frac{1}{5}\sin 5x + C$

C

10. $x\underline{i} + 5\underline{j} + 7\underline{k} \perp -3\underline{i} + 2\underline{j} - \underline{k}$

$\Rightarrow \quad x.(-3) + 5.2 + 7.(-1) = 0$

$-3x + 10 - 7 = 0$

$-3x + 3 = 0$

$3x = 3$

$x = 1$

B

11. $f(x) = \cos x \quad g(x) = x + \frac{\pi}{6}$

$f(g(\frac{\pi}{6})) = f(\frac{\pi}{6} + \frac{\pi}{6})$

$= f(\frac{\pi}{3})$

$= \cos\frac{\pi}{3}$

$= \frac{1}{2}$ D

12. $f(x) = \frac{1}{\sqrt[5]{x}}$

$= x^{-1/5}$

$f'(x) = -\frac{1}{5}x^{-6/5}$ A

13. $y = ax^2 + bx + c$

$a > 0 \Rightarrow$ minimum T.P.

$b^2 - 4ac > 0 \Rightarrow$ two real and distinct roots

B

14. $\int_{-2}^{2} (14 - x^2 - (2x^2 + 2))\, dx$

$= \int_{-2}^{2} (12 - 3x^2)\, dx$

C

15. $f'(x) = x^2 - 9$

$f'(1) = (1)^2 - 9$

$= -8 \Rightarrow$ decreasing

$f'(-3) = (-3)^2 - 9$

$= 0 \Rightarrow$ stationary

Only statement (2) correct

C

16. Second root $= 5 \Rightarrow t = -5$

y-intercept $= k.(-1)^2.(-5)$

$\Rightarrow -5k = 10$

$k = -2$

A

17. $s(t) = t^2 - 5t + 8$

$s'(t) = 2t - 5$

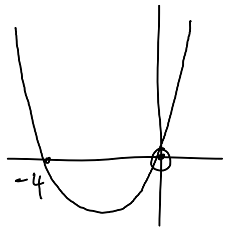
$s'(3) = 2(3) - 5$

$= 1$

B

18. $x^2 + 4x > 0$

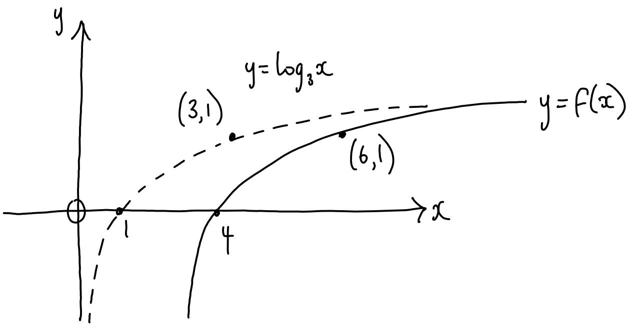
$x(x+4) > 0$



$x < -4$ and $x > 0$

B

19.



$f(x) = \log_3(x-3)$

C

20. (12, 7)

A

PART B

21. (a) $A(4,0)$ $B(-4,16)$ $C(18,20)$

$Q(11,10)$

$$m_{BQ} = \frac{10-16}{11-(-4)} = \frac{-6}{15} = -\frac{2}{5}$$

$$y - b = m(x-a)$$
$$y - 10 = -\frac{2}{5}(x-11)$$
$$5y - 50 = -2x + 22$$
$$2x + 5y - 72 = 0$$

(b) $T(6,12)$

$$2(6) + 5(12) - 72 = 0$$
$$12 + 60 - 72 = 0$$

LHS $\equiv$ RHS

$\Rightarrow$ T lies on BQ

(c) $\vec{BT} = \begin{pmatrix} 10 \\ -4 \end{pmatrix}$ $\quad$ $\vec{TQ} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

$$= 2\begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$\Rightarrow$ T divides BQ in ratio 2:1

22 (a) (i) $f(x) = 2x^3 + x^2 - 8x + 5$

$$\begin{array}{c|cccc} x=1 & 2 & 1 & -8 & 5 \\ & & 2 & 3 & -5 \\ \hline & 2 & 3 & -5 & \boxed{0} \end{array}$$

$\Rightarrow (x-1)$ is a factor

(ii) Quotient is $2x^2 + 3x - 5$

$$f(x) = (x-1)(2x^2+3x-5)$$
$$= (x-1)(2x+5)(x-1)$$
$$= (2x+5)(x-1)^2$$

(b) $$2x^2 + x^2 - 8x + 5 = 0$$

$$(2x+5)(x-1)^2 = 0$$

$$\Rightarrow \quad 2x+5 = 0 \quad \text{or} \quad x-1 = 0$$

$$2x = -5 \qquad x = 1$$

$$x = -\frac{5}{2}$$

roots at $x = -\frac{5}{2}, 1$

(c) $y = 2x^3 + x^2 - 6x + 2$ $\qquad$ $y = 2x - 3$ is a tangent

For points of intersection:

$$2x^3 + x^2 - 6x + 2 = 2x - 3$$

$$2x^2 + x^2 - 8x + 5 = 0$$

$\Rightarrow$ intersects at $x = -\frac{5}{2}, 1$

(Now check gradient of curve at each point.)

$$\frac{dy}{dx} = 6x^2 + 2x - 6$$

when $x = -\frac{5}{2}$,

$$\frac{dy}{dx} = 6\left(-\frac{5}{2}\right)^2 + 2\left(\frac{5}{2}\right) - 6$$

$$= 6.\frac{25}{4} - \frac{10}{2} - 6$$

$$= \frac{75}{2} - \frac{10}{2} - \frac{12}{2}$$

$$= -\frac{97}{2}$$

this is not the gradient of tangent $y = 2x - 3$

when $x = 1$,

$$\frac{dy}{dx} = 6(1)^2 + 2(1) - 6$$

$$= 6 + 2 - 6$$

$$= 2$$

this is the gradient of the tangent $y = 2x - 3$

$$y = 2(1) - 3$$

$$y = -1$$

$\Rightarrow G(1, -1)$

(d) $$y = 2\left(-\frac{5}{2}\right) - 3$$

$$y = -5 - 3$$

$$y = -8$$

$\Rightarrow H\left(-\frac{5}{2}, -8\right)$

23. (a) (i) $3x - 2y = 0$

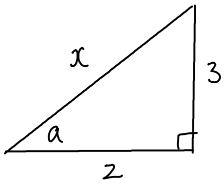
$$2y = 3x$$

$$y = \frac{3}{2}x$$

$$\Rightarrow m = \frac{3}{2} \quad \& \quad m = \tan\theta$$

$$\Rightarrow \tan a = \frac{3}{2}$$

(ii)



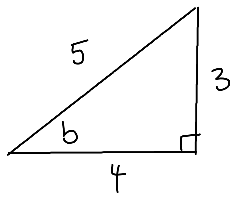
$$x = \sqrt{3^2+2^2}$$

$$x = \sqrt{13} \qquad \sin a = \frac{3}{\sqrt{13}}$$

(b) $3x - 4y = 0$

$$4y = 3x$$

$$y = \frac{3}{4}x$$



$$\Rightarrow m = \frac{3}{4} \quad \& \quad m = \tan\theta$$

$$\Rightarrow \tan b = \frac{3}{4}$$

$$\sin b = \frac{3}{5}$$

$$\cos b = \frac{4}{5}$$

(c) (i) $\sin(a-b) = \sin a \cos b - \cos a \sin b$

$$= \frac{3}{\sqrt{13}} \cdot \frac{4}{5} - \frac{2}{\sqrt{13}} \cdot \frac{3}{5}$$

$$= \frac{12}{5\sqrt{13}} - \frac{6}{5\sqrt{13}}$$

$$= \frac{6}{5\sqrt{13}} \quad \text{or} \quad \frac{6\sqrt{13}}{65} \text{ with rational denominator}$$

(ii) $\sin(b-a) = -\dfrac{6}{5\sqrt{13}} \quad \left(\text{or } -\dfrac{6\sqrt{13}}{65}\right)$