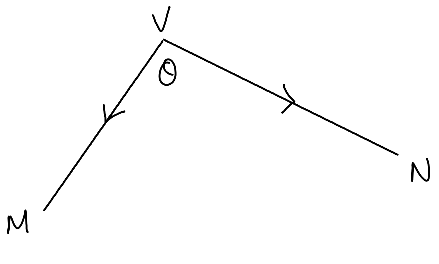
1 (a) $M(0,1,0)$ $N(4,2,2)$

(b) $V(0,2,3)$

$\vec{VM} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$ $\vec{VN} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$

(c)



$|\vec{VM}| = \sqrt{0^2 + (-1)^2 + (-3)^2}$
$= \sqrt{10}$

$|\vec{VN}| = \sqrt{4^2 + 0^2 + (-1)^2}$
$= \sqrt{17}$

$\vec{VM}.\vec{VN} = 0.4 + (-1).0 + (-3).(-1)$
$= 3$

$\underline{a}.\underline{b} = |\underline{a}||\underline{b}|\cos\theta$

$3 = \sqrt{10}.\sqrt{17}\cos\theta$

$\cos\theta = \frac{3}{\sqrt{170}}$

$\theta = \cos^{-1}\left(\frac{3}{\sqrt{170}}\right)$

$\theta = 76.7^\circ$

2. (a) $12\cos x^\circ - 5\sin x^\circ$

$k\cos(x+a)^\circ = k\cos x^\circ\cos a^\circ - k\sin x^\circ\sin a^\circ$

$k\cos a^\circ = 12$

$k\sin a^\circ = 5$

$k^2 = 12^2 + 5^2$

$k^2 = 144 + 25$

$k^2 = 169$

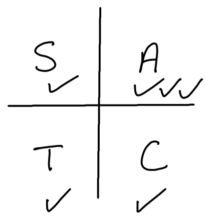
$k = 13$

$\tan a^\circ = \frac{5}{12}$

acute angle $a^\circ = \tan^{-1}\left(\frac{5}{12}\right)$
$= 22.6^\circ$

a° is in 1st quadrant

$\Rightarrow a^\circ = 22.6^\circ$



(b) $12\cos x^\circ - 5\sin x^\circ = 13\cos(x + 22.6)^\circ$

Maximum value is 13

Minimum value is -13

(c) $y = \cos x^\circ$ maximum at 0° and 360°

$y = 13\cos(x + 22.6)^\circ$ maximum at $0 - 22.6^\circ = -22.6^\circ$

and $360 - 22.6^\circ = 337.4^\circ$

$y = \cos x^\circ$ minimum at 180°

$y = 13\cos(x + 22.6)^\circ$ minimum at $180 - 22.6^\circ = 157.4^\circ$

Max at 337.4° and min at 157.6 in the interval $0 \leqslant x \leqslant 360$

3. (a) (i) $x^2 + y^2 + 14x + 4y - 19 = 0$ $\quad y = 3 - x$

for points of intersection:

$$x^2 + (3-x)^2 + 14x + 4(3-x) - 19 = 0$$

$$x^2 + 9 - 6x + x^2 + 14x + 12 - 4x - 19 = 0$$

$$2x^2 + 4x + 2 = 0$$

$$x^2 + 2x + 1 = 0$$

$$(x+1)(x+1) = 0$$

$x = -1$ twice

one solution $\Rightarrow$ line is tangent to circle

(ii) when $x = -1$, $y = 3 - (-1)$

$y = 4$

$\Rightarrow P(-1, 4)$

(b) Larger circle

$C_1(-7,-2)$ $\quad r_1 = \sqrt{(-7)^2 + (-2)^2 - (-19)}$

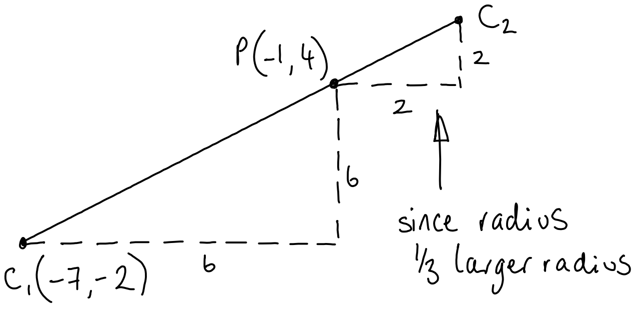
$r_1 = \sqrt{49 + 4 + 19}$

$r_1 = \sqrt{72}$

$r_1 = \sqrt{9 \times 8}$

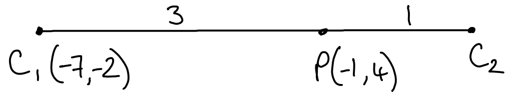
$r_1 = 3\sqrt{8}$ (deliberately not fully simplified as $r_1 = 3r_2$)

$\Rightarrow r_2 = \sqrt{8}$



$\Rightarrow C_2(1,6)$

OR/ use 'points dividing lines' technique



$$\overrightarrow{C_1C_2} = \frac{4}{3}\overrightarrow{C_1P}$$

$$= \frac{4}{3}\begin{pmatrix}6\\6\end{pmatrix}$$

$$\overrightarrow{C_1C_2} = \begin{pmatrix}8\\8\end{pmatrix}$$

$$\Rightarrow C_2(1,6)$$

$\Rightarrow$ Equation of smaller circle is $(x-1)^2 + (y-6)^2 = 8$

4. $$2\cos 2x - 5\cos x - 4 = 0 \qquad 0 \leq x < 2\pi$$

$$2(2\cos^2 x - 1) - 5\cos x - 4 = 0$$

$$4\cos^2 x - 2 - 5\cos x - 4 = 0$$

$$4\cos^2 x - 5\cos x - 6 = 0$$

$$(4\cos x + 3)(\cos x - 2) = 0$$

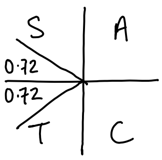
$$4x^2 - 5x - 6 = 0$$
$$(4x+3)(x-2) = 0$$

$$\begin{pmatrix}4 & 3\\1 & -2\end{pmatrix}$$
$$3 \quad -8$$

$\Rightarrow 4\cos x + 3 = 0$ or $\cos x - 2 = 0$

$\cos x = -\frac{3}{4}$ $\qquad$ $\cos x = 2$

acute angle $x = 0.72$ rads $\qquad$ undefined



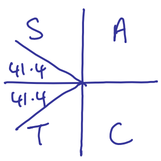
$x = 2.42,\ 3.86$ radians

$\pi - 0.72 = 2.42$ rads

$\pi + 0.72 = 3.86$ rads

WORKING IN DEGREES

acute angle $x° = \cos^{-1}\left(\frac{3}{4}\right)$
$= 41.4°$



$180 - 41.4 = 138.6$

$180 + 41.4 = 221.4$

$x° = 138.6°,\ 221.4°$

$x = 138.6 \cdot \frac{\pi}{180},\ 221.4 \cdot \frac{\pi}{180}$

$x = 2.42,\ 3.86$ radians

5. (a) (i) If $TP = x$ then x-coordinate of Q is x

$\Rightarrow$ y-coordinate of Q is $y = 10 - x^2$

T is y-intercept of curve $y = \frac{2}{5}(10 - x^2)$

$\Rightarrow$ $T(0, 4)$

$$\Rightarrow \quad PQ = 10 - x^2 - 4$$
$$= 6 - x^2$$

(ii) Area of $PQRS = 2x(6 - x^2)$

$$\Rightarrow \quad A(x) = 12x - 2x^3$$

(b) For maximum area, $A'(x) = 0$

$$\Rightarrow \quad 12 - 6x^2 = 0$$
$$6x^2 = 12$$
$$x^2 = 2$$
$$x = \sqrt{2}$$

($-\sqrt{2}$ discarded for length)

x	1	$\sqrt{2}$	2
$A'(x)$	$+$	0	$-$
shape	/	—	\

Max TP at $x = \sqrt{2}$

$$A(\sqrt{2}) = 12\sqrt{2} - 2(\sqrt{2})^3$$
$$= 11 \cdot 3$$

Maximum area is $11 \cdot 3$ units^2

6. (a) $y = (2x-9)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(2x-9)^{-1/2} \cdot 2$

$= (2x-9)^{-1/2}$

$= \frac{1}{\sqrt{(2x-9)}}$

when $x = 9$,

$m = \frac{1}{\sqrt{(2(9)-9)}}$

$m = \frac{1}{\sqrt{9}}$

$m = \frac{1}{3}$

$y = \sqrt{2(9)-9}$

$y = \sqrt{9}$

$y = 3$

$y - b = m(x-a)$

$y - 3 = \frac{1}{3}(x-9)$

$y - 3 = \frac{1}{3}x - 3$

$y = \frac{1}{3}x$

(b) $\sqrt{(2x-9)} = 0$

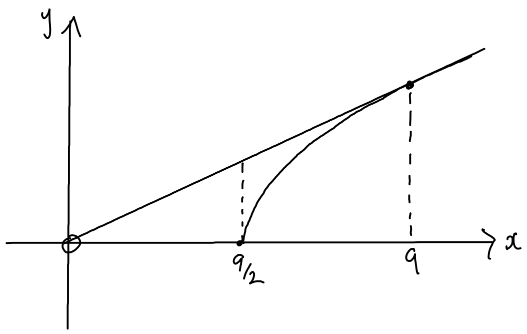
$2x - 9 = 0$

$2x = 9$

$x = \frac{9}{2}$

$A(\frac{9}{2}, 0)$

(c)



Either split area into two sections and integrate line between 0 & 9/2 then between curves 9/2 to 9, add your answers to find shaded area.

Or integrate line between 0 & 9 and curve between 9/2 and 9 then subtract.

The latter method is on next page.

$$A_1 = \int_0^9 \frac{1}{3}x \, dx$$

$$A_1 = \left[\frac{1}{3} \cdot \frac{x^2}{2} \right]_0^9$$

$$A_1 = \left[\frac{x^2}{6} \right]_0^9$$

$$A_1 = \left(\frac{(9)^2}{6} \right) - \left(\frac{(0)^2}{6} \right)$$

$$A_1 = \frac{81}{6}$$

$$A_1 = \frac{27}{2}$$

$$A_2 = \int_{9/2}^{9} (2x-9)^{1/2} \, dx$$

$$A_2 = \left[\frac{(2x-9)^{3/2}}{\frac{3}{2} \cdot 2} \right]_{9/2}^{9}$$

$$A_2 = \left[\frac{1}{3} \sqrt{(2x-9)^3} \right]_{9/2}^{9}$$

$$A_2 = \left(\frac{1}{3} \left(\sqrt{(2(9)-9)^3} \right) \right) - \left(\frac{1}{3} \sqrt{(2(\tfrac{9}{2})-9)^3} \right)$$

$$A_2 = \frac{1}{3} \sqrt{9^3} - \frac{1}{3} \sqrt{0^3}$$

$$A_2 = \frac{1}{3} \cdot 27$$

$$A_2 = 9$$

$$\text{Shaded Area} = \frac{27}{2} - 9$$

$$= \frac{27}{2} - \frac{18}{2}$$

$$= \frac{9}{2} \text{ units}^2$$

7. (a)

$$\log_4 x = p$$

$$\Rightarrow 4^p = x$$

$$4 = 16^{1/2} \quad \text{so} \quad (16^{1/2})^p = x$$

$$16^{\frac{1}{2}p} = x$$

$$\log_{16} x = \tfrac{1}{2} p$$

(b)

$$\log_3 x + \log_9 x = 12$$

If $\log_3 x = Q$

then $\log_9 x = \frac{1}{2} Q$

$$\Rightarrow Q + \tfrac{1}{2} Q = 12$$

$$Q = 8$$

$$\Rightarrow \log_3 x = 8$$

$$x = 3^8$$