

2011 HIGHER PAPER 1 SOLUTIONS

1. $p = \begin{pmatrix} 2 \\ 5 \\ -7 \end{pmatrix}$ $q = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ $r = \begin{pmatrix} -4 \\ 2 \\ 0 \end{pmatrix}$

$$2p - q - \frac{1}{2}r$$

$$= 2 \begin{pmatrix} 2 \\ 5 \\ -7 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} -4 \\ 2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 10 \\ -14 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ 9 \\ -13 \end{pmatrix}$$

(C)

2. $3y + 2x = 6$

$$3y = -2x + 6$$

$$y = -\frac{2}{3}x + 2$$

$$m = -\frac{2}{3}$$

(B)

3. $y = f(x+2) - 1$

shift 2 left, 1 down

$$(1, 2) \rightarrow (-1, 1)$$

$$(-2, -3) \rightarrow (-4, -4)$$

(D)

4. $y = x^3 - 2x$ $(2, 4)$

$$\frac{dy}{dx} = 3x^2 - 2$$

when $x = 2$

$$m = 3(2)^2 - 2$$

$$m = 10$$

(D)

5. $x^2 - 8x + 7$
 $= (x-4)^2 - 16 + 7$
 $= (x-4)^2 - 9$

(A)

6. $m_{cp} = -2$ $P(2, -3)$
 $m_T = \frac{1}{2}$ as $m_1 m_2 = -1$
 for \perp lines

$y - b = m(x - a)$

$y + 3 = \frac{1}{2}(x - 2)$

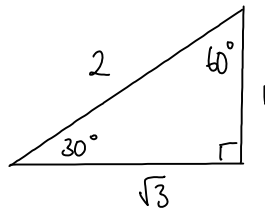
(C)

7. $x = 1$

1	-1	1	3
	1	0	1
1	0	1	4

(D)

8. $m = \tan 30^\circ$
 $m = \frac{1}{\sqrt{3}}$



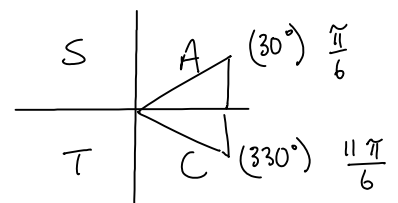
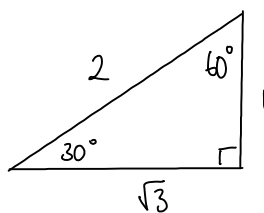
(A)

9. $b^2 - 4ac = 23$
 $b^2 - 4ac > 0$ so roots are real

$\sqrt{23}$ is irrational so roots are irrational

(B)

10. $2 \cos x = \sqrt{3}$
 $\cos x = \frac{\sqrt{3}}{2}$
 $x = \frac{\pi}{6}$



(D)

$$11. \int (4x^{1/2} + x^{-3}) dx$$

$$= \frac{4x^{3/2}}{\frac{3}{2}} + \frac{x^{-2}}{-2} + C$$

$$= \frac{2}{3} \cdot 4x^{3/2} - \frac{1}{2}x^{-2} + C$$

$$= \frac{8}{3}x^{3/2} - \frac{1}{2}x^{-2} + C$$

(D)

$$12. \sin(p+q) = \sin p \cos q + \cos p \sin q$$

$$= \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{3} + \frac{1}{\sqrt{5}} \cdot \frac{2}{3}$$

$$= \frac{2}{3} + \frac{2}{3\sqrt{5}}$$

(C)

$$13. f(x) = 4 \sin 3x$$

$$f'(x) = 4 \cos 3x \cdot 3$$

$$= 12 \cos 3x$$

$$f'(0) = 12 \cos 3(0)$$

$$= 12$$

(C)

$$14. p \cdot q = |p| \cdot |q| \cos \theta$$

$$= 3 \cdot 3 \cdot \cos 60^\circ$$

$$= 9 \cdot \frac{1}{2}$$

$$= \frac{9}{2}$$

(B)

15. $S(-4, 5, 1)$ $T(-16, -4, 16)$ $U(-24, -10, 26)$

$$\begin{aligned}\vec{ST} &= \begin{pmatrix} -12 \\ -9 \\ 15 \end{pmatrix} & \vec{TU} &= \begin{pmatrix} -8 \\ -6 \\ 10 \end{pmatrix} \\ &= 3 \begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix} & &= 2 \begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix}\end{aligned}$$

$$ST : TU = 3 : 2$$

(B)

16. $\int \frac{1}{3x^4} dx$

$$\begin{aligned}&= \int \frac{1}{3} x^{-4} dx \\ &= \frac{1}{3} \frac{x^{-3}}{-3} + C \\ &= \frac{x^{-3}}{-9} + C \\ &= \frac{-1}{9x^3} + C\end{aligned}$$

(A)

17. Roots at $x = -1, 0, 2$

$$\Rightarrow \text{factors } (x+1), x, (x-2)$$

$$\Rightarrow y = kx(x+1)(x-2)$$

sub in (1, 2)

$$2 = k \cdot 1 \cdot 2 \cdot (-1)$$

$$2 = -2k$$

$$k = -1$$

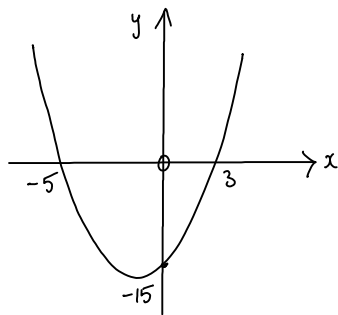
$$\Rightarrow y = -x(x+1)(x-2)$$

(A)

18. $f(x) = (x-3)(x+5)$

$$= x^2 + 2x - 15$$

$$x^2 +ve \Rightarrow \cup$$



(C)

$$f(x) > 0 \text{ when } x < -5 \text{ and } x > 3$$

19. $\log_3 y = x \Leftrightarrow y = 3^x$

(C)

20. $g(x) = \sin^2 \sqrt{x-2}$

$\sqrt{x-2}$ cannot be negative so $x \geq 2$

$\sin^2 \sqrt{x-2} = (\sin \sqrt{x-2})^2$ cannot be negative, $\max = 1$, $\min = 0$

$$x \geq 2, \quad 0 \leq g(x) \leq 1 \quad \text{(D)}$$

21. (a) $B(7, 12)$ $D(2, -3)$

$$m_{BD} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-3 - 12}{2 - 7}$$

$$= \frac{-15}{-5}$$

$$y - b = m(x - a)$$

$$y - 12 = 3(x - 7)$$

$$y - 12 = 3x - 21$$

$$y = 3x - 9$$

$$m_{BD} = 3$$

(b) $x + 3y = 23$

$$y = 3x - 9$$

for pts of intersection $y = y$

$$x + 3(3x - 9) = 23$$

$$x + 9x - 27 = 23$$

$$10x = 50$$

$$x = 5$$

when $x = 5$,

$$y = 3(5) - 9$$

$$y = 6$$

$$E(5, 6)$$

(c) (i) $A(-1, 8)$ $B(7, 12)$

$$M(3, 10)$$

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{12 - 8}{7 - (-1)}$$

$$= \frac{4}{8}$$

$$= \frac{1}{2}$$

$$m_{\perp} = -2 \text{ as } m_1 m_2 = -1 \text{ for } \perp \text{ lines}$$

$$y - b = m(x - a)$$

$$y - 10 = -2(x - 3)$$

$$y - 10 = -2x + 6$$

$$y = -2x + 16$$

(ii) when $x = 5$

$$y = -2(5) + 16$$

$$y = 6$$

\Rightarrow line passes through E

22. (a) $f(x) = (x-2)(x^2+1)$

(i) on x-axis, $f(x) = 0$

$$(x-2)(x^2+1) = 0$$

$$x = 2 \quad \left(\begin{array}{l} x^2+1 = 0 \\ \text{no solutions} \end{array} \right)$$

$$(2, 0)$$

(ii) on y-axis $x = 0$

$$f(0) = (-2) \cdot 1$$

$$= -2$$

$$(0, -2)$$

(b) $f(x) = (x-2)(x^2+1)$

$$= x^3 - 2x^2 + x - 2$$

$$f'(x) = 3x^2 - 4x + 1$$

for stat pts $f'(x) = 0$

$$3x^2 - 4x + 1 = 0$$

$$(3x-1)(x-1) = 0$$

$$x = \frac{1}{3}, 1$$

stat pts at $(\frac{1}{3}, -\frac{50}{27})$ & $(1, -2)$

x	\rightarrow	$\frac{1}{3}$	\rightarrow	1	\rightarrow	
$f'(x)$		$+$	0	$-$	0	$+$
shape		$/$	$-$	\backslash	$-$	$/$

Max T.P. at $(\frac{1}{3}, -\frac{50}{27})$

Min T.P. at $(1, -2)$

$$f(\frac{1}{3}) = (\frac{1}{3})^3 - 2(\frac{1}{3})^2 + (\frac{1}{3}) - 2$$

$$= \frac{1}{27} - \frac{2}{9} + \frac{1}{3} - 2$$

$$= \frac{1}{27} - \frac{6}{27} + \frac{9}{27} - \frac{54}{27}$$

$$= -\frac{50}{27}$$

$$f(1) = (1)^3 - 2(1)^2 + (1) - 2$$

$$= -2$$

$$f'(0) = 3(0)^2 - 4(0) + 1$$

$$= 1$$

$$f'(\frac{1}{2}) = 3(\frac{1}{2})^2 - 4(\frac{1}{2}) + 1$$

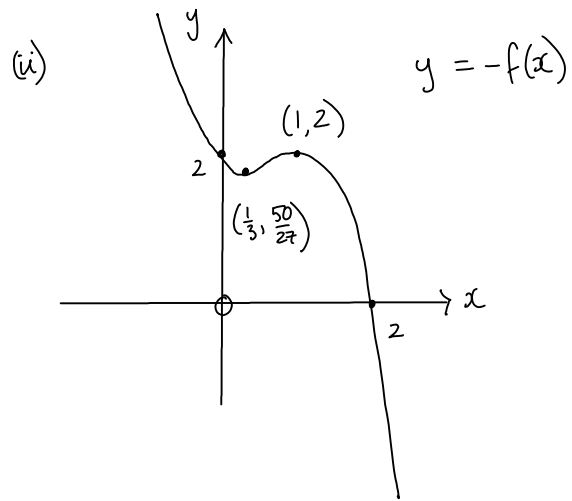
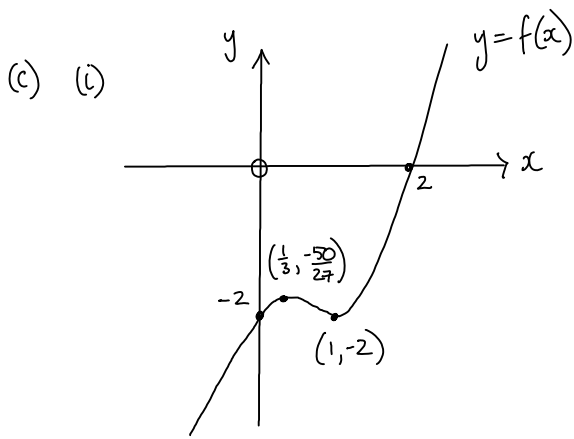
$$= \frac{3}{4} - 2 + 1$$

$$= -\frac{1}{4}$$

$$f'(2) = 3(2)^2 - 4(2) + 1$$

$$= 12 - 8 + 1$$

$$= 5$$



23. (a) $\cos 2x^\circ - 3\cos x^\circ + 2 = 0$ $0 \leq x < 360$

$$2\cos^2 x^\circ - 1 - 3\cos x^\circ + 2 = 0$$

$$2\cos^2 x^\circ - 3\cos x^\circ + 1 = 0$$

$$(2\cos x^\circ - 1)(\cos x^\circ - 1) = 0$$

either $2\cos x^\circ - 1 = 0$ or $\cos x^\circ - 1 = 0$

$$\cos x^\circ = \frac{1}{2}$$

$$x^\circ = 60^\circ, 300^\circ$$

$$x^\circ \in \{0^\circ, 60^\circ, 300^\circ\}$$

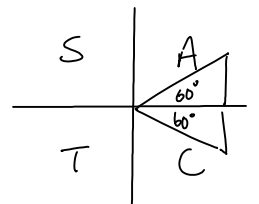
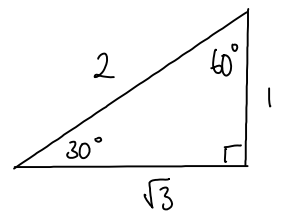
$$\cos x^\circ = 1$$

$$x^\circ = 0^\circ, 360^\circ$$

(discard 360° as out with domain)

$$2x^2 - 3x + 1$$

$$(2x-1)(x-1)$$



(b) $\cos 4x^\circ - 3\cos 2x^\circ + 2 = 0$

$$\cos 2(2x^\circ) - 3\cos(2x^\circ) + 2 = 0$$

From (a) $2x^\circ = 60^\circ, 300^\circ$ $2x^\circ = 0^\circ, 360^\circ$

$$x^\circ = 30^\circ, 150^\circ, 210^\circ, 330^\circ \quad x^\circ = 0^\circ, 180^\circ, 360^\circ$$

$$x^\circ \in \{0^\circ, 30^\circ, 150^\circ, 180^\circ, 210^\circ, 330^\circ\}$$