2011 HIGHER PAPER 2 SOLUTIONS

1. (a) $B(4, 4, 0)$

(b) $\vec{DB} = \begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$ $\quad \vec{DM} = \begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$

(c)

$$\vec{DB} \cdot \vec{DM} = |\vec{DB}||\vec{DM}| \cos\theta$$

$$32 = \sqrt{44} \cdot \sqrt{40} \cos\theta$$

$$\cos\theta = \frac{32}{\sqrt{44}\sqrt{40}}$$

$$\theta = \cos^{-1}\left(\frac{32}{\sqrt{44}\sqrt{40}}\right)$$

$$\theta = 40{\cdot}3^\circ$$

$$\angle BDM = 40{\cdot}3^\circ$$

$$\vec{DB} \cdot \vec{DM} = 2.0 + 2(-2) + (-6)(-6)$$
$$= 32$$

$$|\vec{DB}| = \sqrt{2^2 + 2^2 + (-6)^2}$$
$$= \sqrt{4 + 4 + 36}$$
$$= \sqrt{44}$$

$$\vec{DM} = \sqrt{0^2 + (-2)^2 + (-6)^2}$$
$$= \sqrt{0 + 4 + 36}$$
$$= \sqrt{40}$$

2 (a) $f(x) = x^3 - 1 \quad g(x) = 3x + 1$

$$g(f(x)) = g(x^3 - 1)$$
$$= 3(x^3 - 1) + 1$$
$$= 3x^3 - 2$$

(b)

$$g(f(x)) + xh(x)$$
$$= 3x^3 - 2 + x(4x - 5)$$
$$= 3x^3 + 4x^2 - 5x - 2$$

(c) (i)

$$x = 1 \;\left|\; \begin{array}{cccc} 3 & 4 & -5 & -2 \\ & 3 & 7 & 2 \\ \hline 3 & 7 & 2 & \boxed{0} \end{array}\right.$$

$\Rightarrow (x - 1)$ is a factor

(ii) Quotient is $3x^2 + 7x + 2$

$$g(f(x)) + xh(x) = (x-1)(3x^2 + 7x + 2)$$
$$= (x-1)(3x+1)(x+2)$$

(d)

$$(x-1)(3x+1)(x+2) = 0$$

$$x = -2, -\tfrac{1}{3}, 1$$

3(a) $u_{n+1} = -\frac{1}{2}u_n \qquad u_0 = -16$

$u_1 = -\frac{1}{2}(-16) \qquad u_2 = -\frac{1}{2}(8)$

$= 8 \qquad = -4$

(b) $4, 5, 7, 11$

$v_2 = pv_1 + q \qquad v_3 = pv_2 + q$

$5 = 4p + q \qquad 7 = 5p + q$

$q = 5 - 4p \qquad q = 7 - 5p$

$\Rightarrow \quad 5 - 4p = 7 - 5p$

$p = 2$

when $p = 2$,

$q = 5 - 4(2)$

$q = -3$

(c) (i) $u_{n+1} = -\frac{1}{2}u_n$

a limit exists as $-1 < -\frac{1}{2} < 1$

at the limit $u_{n+1} = u_n = L$

$L = -\frac{1}{2}L$

$\frac{3}{2}L = 0$

$L = 0$

The limit is 0

(ii) $v_{n+1} = 2v_n - 3$

for a limit to exist

$-1 < m < 1$

a limit does not exist

as $2 > 1$

4. $$A_1 = \int_{-2}^{0} \left(x^3 - x^2 - 4x + 4 - (2x+4)\right) dx$$

$$= \int_{-2}^{0} \left(x^3 - x^2 - 6x\right) dx$$

$$= \left[\frac{x^4}{4} - \frac{x^3}{3} - \frac{6x^2}{2}\right]_{-2}^{0}$$

$$= \left[\frac{x^4}{4} - \frac{x^3}{3} - 3x^2\right]_{-2}^{0}$$

$$= \left(\frac{(0)^4}{4} - \frac{(0)^3}{3} - 3(0)^2\right) - \left(\frac{(-2)^4}{4} - \frac{(-2)^3}{3} - 3(-2)^2\right)$$

$$= 0 - \left(\frac{16}{4} - \frac{(-8)}{3} - 3.4\right)$$

$$= -\left(4 + \frac{8}{3} - 12\right)$$

$$= 8 - \frac{8}{3}$$

$$= \frac{24}{3} - \frac{8}{3}$$

$$= \frac{16}{3} \text{ units}^2$$

$$A_2 = \int_{0}^{3} \left(2x + 4 - (x^3 - x^2 - 4x + 4)\right) dx$$

$$= \int_{0}^{3} \left(6x + x^2 - x^3\right) dx$$

$$= \left[3x^2 + \frac{x^3}{3} - \frac{x^4}{4}\right]_{0}^{3}$$

$$= \left(3(3)^2 + \frac{(3)^3}{3} - \frac{(3)^4}{4}\right) - 0$$

$$= 27 + \frac{27}{3} - \frac{81}{4}$$

$$= 36 - \frac{81}{4}$$

$$= \frac{144}{4} - \frac{81}{4}$$

$$A_2 = \frac{63}{4} \text{ units}^2$$

$$\text{Total Area} = \frac{16}{3} + \frac{63}{4}$$

$$= \frac{64}{12} + \frac{189}{12}$$

$$= \frac{253}{12}$$

$$= 21\tfrac{1}{12} \text{ units}^2$$

5.

$y = kx^n$

$\log y = \log kx^n$

$\log y = \log k + \log x^n$

$\log y = n\log x + \log k$

$Y = mX + c$

$n = m$

$n = \frac{1}{2}$

$c = \log k$

$5 = \log_2 k$

$k = 2^5$

$k = 32$

$(0,5) \quad (4,7)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$c = 5$

$m = \frac{7-5}{4-0}$

$m = \frac{2}{4}$

$m = \frac{1}{2} \Rightarrow y = mx + c$

$y = \frac{1}{2}x + 5$

$y = 32x^{1/2}$

6 (a)

$3\underline{\sin x} - 5\underline{\underline{\cos x}}$

$R\sin(x+a) = R\underline{\sin x}\cos a + R\underline{\underline{\cos x}}\sin a$

$R\cos a = 3$

$R\sin a = -5$

$R^2 = (3)^2 + (-5)^2$

$R^2 = 9 + 25$

$R^2 = 34$

$R = \sqrt{34}$

$\frac{R\sin a}{R\cos a} = \tan a$

$\tan a = \frac{-5}{3}$

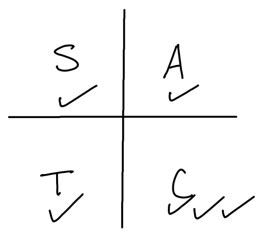
acute angle $= 1.03$ rads

a is in 4^{th} quadrant

$\Rightarrow a = 2\pi - 1.03$

$a = 5.3$ rads

$(59^\circ \times \frac{\pi}{180} = 1.03 \text{ rads})$



$\Rightarrow 3\sin x - 5\cos x = \sqrt{34}\sin(x + 5.3)$

6(b)

$$\int_0^t (3\cos x + 5\sin x)\,dx = 3$$

$$\left[3\sin x - 5\cos x\right]_0^t = 3$$

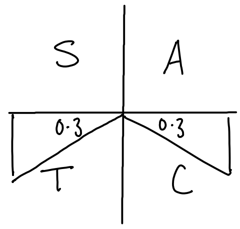
$$\left[\sqrt{34}\sin(x+5.3)\right]_0^t = 3$$

$$\sqrt{34}\sin(t+5.3) - \sqrt{34}\sin(5.3) = 3$$

$$\sqrt{34}\sin(t+5.3) + 4.9 = 3$$

$$\sqrt{34}\sin(t+5.3) = -1.9$$

$$\sin(t+5.3) = \frac{-1.9}{\sqrt{34}}$$



acute angle $= 0.3$ rads

$\pi + 0.3 = 3.4$ rads

$2\pi - 0.3 = 6.0$ rads

$$t + 5.3 = 3.4,\ 6.0$$

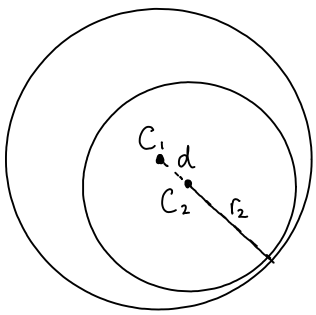
$$t = -1.9,\ 0.7$$

$-1.9 + 2\pi = 4.4$

$$t = 0.7,\ 4.4 \text{ rads}$$

$\Rightarrow \quad t = 0.7$ to 1 d.p.

7.



$C_1(-1,1)$ $r_1 = 11$ $\quad C_2(2,-3)$

Distance between centres, $d = \sqrt{(2-(-1))^2 + ((-3)-1)^2}$

$$d = \sqrt{3^2 + (-4)^2}$$

$$d = \sqrt{25}$$

$$d = 5$$

$\Rightarrow \quad r_2 < 6$

$$\sqrt{(2)^2 + (-3)^2 - p} < 6$$

$$\sqrt{13 - p} < 6$$

$$13 - p < 36$$

$$-p < 23$$

$$p > -23$$

$r_2 = \sqrt{13 - p}$

for C_2 to exist, $p < 13$

$\Rightarrow \quad -23 < p < 13$